

FACULDADE DE CIÊNCIAS E TECNOLOGIA

Presidium of National Academy of Sciences of Azerbaijan

RECOMMENDATION LETTER

for Professor Vagif Sabir ogly Guliyev, Doctor of Science in Physics and Mathematics, principal scientific researcher of the Institute of Mathematics and Mechanics of NAS of Azerbaijan

March 16, 2012

Institute of Mathematics and Mechanics of Института Математики of NAS of Azerbaijan proposed the candidature of Professor Vagif Sabir ogly Guliyev, Doctor of Science in Physics and Mathematics for election as a corresponding member of NAS of Azerbaijan.

Without any reservation, I support the candidature of Professor V.S.Guliyev, the author of a series of fundamental research in the area of mathematical analysis.

Professor Vagif Guliyev is an internationally known mathematician. In such areas as harmonic analysis, functional analysis, function theory and partial differential equations, mathematical schools of Azerbaijan are known to have scientists of the worldwide level. Undoubtedly, Professor Vagif Guliyev is one of them. He is the leader of a scientific school known in Azerbaijan and abroad. He was a supervisor of 10 PhD students (Candidates of Science). Three of them were awarded by INTAS grants for young researchers.

I am familiar with Professor V.Guliyev from 1983, when I was an official opponent in the defense of his PhD Theses (degree of Candidate of Sciences) "Investigation of anisotropic singular operator", which took place on June 22 of 1983. In the sequel, we often met at Seminar of the Department of Function Theory of Steklov Institute of Mathematics of Academy of Sciences in Moscow, in particular, during presentations of talks of Vagif Guliyev at this seminar on embedding and approximation theorems for Nikolsky-Besov spaces on Lie homogeneous groups, and also on various conferences, both in the former SU and abroad. I keep in memory numerous discussions with him of various mathematical problems; these discussions always impressed me both by his keenness on the subject and an ability to quickly eneter into new areas and obtain interesting results there.

Mathematical tastes and the area of scientific interests of V.S. Guliyev during the subsequent decades were mainly formed by the influence of Department of Function Theory in Steklov Mathematical Institute, where V.S.Guliyev was between 1987 and 1991, first of all in his contacts with Professor P.I.Lizorkin and Professor O.V.Besov, corresponding member of RAN, in this institute. In 1994 Γ in the same institute v.S.Guliyev defended his Doctor degree "Integral operators in the spaces of functions on homogeneous Lies groups and domains in Rⁿ",

to which I wrote an external report. Already in this dissertation V.S.Guliyev obtained an essential advance in the theory of potential type operators and singular operators. Among many important results of this dissertation the following are worthwhile to be mentioned: the theorem on the boundedness of anisotropic singular operators and effectively realizable theorems on Fourier multipliers for Banach-valued functions, which was shown by the author in particular in

application of these theorems on multipliers to Banach-valued Sobolev spaces in \mathbb{R}^n with anisotropic fractional smoothness and Lizorkin-Triebel spaces.

The area of scientific interest and professional experience of Vagif Guliyev is very large. Some of them are the following:

1) two-weighted estimates for potential type and singular operators on Lie homogeneous groups,

2) spaces of holomorphic functions of Besov and Lizorkin-Triebel type on the polydisc in boundary value terms,

3) local and global properties of anisotropic singular integral operators in lebesgue type space s and spaces of smooth Banach-valued вах, а также в пространствах гладких банахо functions on homogeneous groups,

4) direct and converse embedding theorems for weighted spaces of Banach-valued functions on homogeneous groups, including the case of potential weight,

5) analogues of classical operators of harmonic analysis generated by Bessel, Laguerre, Gegenbauer operators and others,

6) *a priori* estimates and regularity properties of solutions to parabolic equations with discontinuous coefficients.

This list may be certainly extended.

Starting from 2000, professor Vagif Guliyev develops a new trend, partially jointly with Professor V.I.Burenkov, perspective in harmonic analysis, related to the study of classical operators in general spaces of Morrey type, which are important in applications in function theory and partial differential equations. I particular, within certain assumptions he obtained criteria of boundedness of fractional maximal operator and Riesz potential operator from a general Morrey type spaces to another such space. The significance of the developed methods consists in the fact that they allow to obtain necessary and sufficient conditions for the boundedness of classes of singular type operators with the subsequent application to demonstration of more precise estimates for solutions to elliptic and parabolic equations in partial derivatives.

The topics of research of Professor Vagif Guliyev belong to intensively developed areas of harmonic analysis and functional analysis, functions theory of many complex variables and applications to partial differential equations of mathematical physics. With the background of high competition in mathematical investigations in these areas, many research priorities are due to Professor V.S.Guliyev, in particular and first of all in the theory of banach-valued functions and generalization of Morrey-type spaces.

Results obtained by Professor Guliyev, are well known both within the former Soviet Union, and also in many countries of Europe and in USA.

Professor Guliyev wrote three monographs and more than 130 papers, published in known scientific journals such as DAN SSSR or RAN (1989, 1991, 1991, 1993, 1994, 1995, 1997, 1998, 1999, 2006, 2007), Proceedings of Steklov Mathematical Institute (1992, 1993, 1993, 1995, 1997, 2003), Siberian Mathematical Journal (2007, 2008, 2009), Integral Transforms and Special Functions (2007, 2008, 2010, 2011), Integral Equations and Operator Theory (2008, 2011), Analysis Mathematica (1995, 1998, 2000, 2008), Mathematical Inequalities and Applications (2003, 2008, 2012), Fractional Calculus and Applied Analysis (2006, 2006, 2008), Journal of Mathematical Analysis and Applications (2007, 2008, 2008, 2009), Potential Analysis (2009, 2011), Complex variables and elliptic equations (2010, 2010, 2012), Georgian Mathematical Journal (1994, 2008, 2012), Journal of Mathematical Inequalities" (2008, 2010, 2011, 2011), Mathematica Scandinavica (2010) и т.д.

Professor V.Guliyev actively attends many international conferences, at some of them we met last years. He is a member of "Society for Analysis, its Applications and Computation" (ISAAC) and presented talks at the 6th Congress of ISAAC in USA (1997), Japan (1999), Germany (2001), Canada (2003), Turkey (2007) and Great Britain (2009). He is a member of Editorial Board of "Eurasian Mathematical Journal".

Results obtained by Professor V.S.Guliyev in the above mentioned areas undoubtedly make him one of the leading experts in these areas on the worldwide level. In February of this year Professor V.Guliyev celebrated 55 years. He is in the prime of life, full of creative plans and will be able to apply his experience and knowledge for formation of scientific specialists of Azerbaijan. All this, as well as all stated above impels me to support the candidature of Professor V.S.Guliyev for election him a corresponding member of national Academy of Sciences of Azerbaijan. By my strong belief, he is a worthy candidate for this election and absolutely merits to be elected a corresponding member of NAS of Azerbaijan.

S.G. Samko Professor of University of Algarve, Portugal Professor of Federal Southern University, Russia Honorary Academician of Academy of Sciences of Higher School of Ukraine